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# BCA 3RD SEMSESTER

## COPUTER ORIENTED NUMERICAL METHOD

## DCA2101

# (set:- I )

1) **Question) Find Taylor's series of the function $f(x) = 3x^5 - 2x^4 + 15x^3 + 13x^2 - 12x - 5$ at point c =2 .**

**Answer :-**

We know Taylor's series expansion of function at the point $x0$ is

$f(x) = f(x0) + f'(x0)(x - x0) + f''(x0)\ 2!\ (x - x0)\ 2 + \cdots + f\ (n)\ (x0)\ n!\ (x - x0)\ n$

So at $x0 = 2$, we have

$f(x) = f(2) + f'(2)(x - 2) + f''(2)\ /2!\ (x - 2)\ 2 + \cdots + f\ (n)\ (2)\ /n!\ (x - 2)$

$n\ldots$ ..(1)

$f(x) = 3x\ 5 - 2x\ 4 + 15x\ 3 + 13x\ 2 - 12x - 5$

$f(2) = 3(32) - 2(16) + 15(8) + 13(4) - 12(2) - 5 = 207$

$f'(x) = 15x\ 4 - 8x\ 3 + 45x\ 2 + 26x - 12$

$f'(2) = 15(16) - 8(8) + 45(4) + 26(2) - 12 = 396$

$f''(x) = 60x\ 3 - 24x\ 2 + 90x + 26$

$f''(2) = 60(8) - 24(4) + 90(2) + 26 = 590$

$f'''(x) = 180x\ 2 - 48x + 90$

$f'''(2) = 180(4) - 48(2) + 90 = 714$

$f\ iv(x) = 360x - 48$

$f\ iv(2) = 360(2) - 48 = 672$

$f\ v\ (x) = 360$

$f\ v\ (2) = 360$

$f\ vi(x) = 0$

$f\ vi(2) = 0$

Putting these values in (1), we get

$f(x) = 207 + 396(x - 2) + 590(x - 2)\ 2\ /\ 2! + 714(x - 2)\ 3\ /\ 3! + 672(x - 2)\ 4\ /\ 4! + 360(x - 2)\ 5\ /\ 5!$

$f(x) = 207 + 396(x - 2) + 295(x - 2)\ 2 + 119(x - 2)\ 3 + 28(x - 2)\ 4 + 3(x - 2)\ 5$

2) **Question) Evaluate $\sqrt{12}$ to four decimal places by Newton's- Raphson formula .**

**Answer :-**

$\sqrt{12}$

Let x- = $\sqrt{12}$

$X^2 = 12$

$X^2 - 12 = 0$

Iterative eqn. for Newton – Raphson method is

$X_{n=l} = x_n - \frac{f(xn)}{f'(xn)}$

$X_{n=l} = x_n - \frac{(x\text{^}2n-12)}{2x}$

$X_l = x_0 - \frac{(xo^2-12)}{2xo}$

$= 3.5 - \frac{[(3.5)^2-12]}{2\ X\ 3.5}$

- ⇨ 3.5-0.0357
- ⇨ 3.4643 first equation

$X_l = x_l - \frac{(x1^2-12)}{2x1}$

X1 → 3.4643

- ⇨ $3.4643 - \frac{((3.4643)^2-12)}{2*3.4643}$
- ⇨ $3.4643 - \frac{1.37449}{6.9286}$
- ⇨ 34643 – 0.19837 = 3.266 *ans*

3) Question) Slove the system of equation by matrix inversion method

X + y + z = 1

X + 2y +3z = 6

X + 3y +4z = 6

Answer :- A x =3

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix}$$

$A^{+} = \frac{a\infty 8}{|A|}$

$|A| \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

→ 1 ( 0 - 9 ) - 1 ( 4 - 3 ) + ( 3 – 2 )

= - 1 – 1 – 1 -1

| A | = 1

$A^{l} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

*Ad-join*

$0^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$A^{l} \begin{bmatrix} -1 & -1 & -1 \\ -1 & -3 & -2 \\ -1 & -2 & -1 \end{bmatrix}$ -> $\begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ -1 & -3 & -2 \\ -1 & -2 & -1 \end{bmatrix} \quad \begin{bmatrix} -11 \\ -31 \\ -19 \end{bmatrix}$$ *ans*

# (set:- II )

**4) Question) From the following table , estimate the number of students who obtained marks bwteen 40 and 45 .**

| Marks | 30-40 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|
| No. of students | 31 | 42 | 35 | 31 |

| X | x | $\Delta$ | $\Delta^2$ | $\Delta^3$ | $\Delta^4$ |
|---|---|---|---|---|---|
| **40** | **31** | **42** | **9** | **-25** | **37** |
| **50** | **73** | **51** | **-16** | **12** | |
| **60** | **124** | **35** | **4** | | |
| **70** | **159** | **31** | | | |
| **80** | **190** | | | | |

Now we need to find the no of students whose marks is less than 45 From the above table we have,

$y_0 = 31$, $\Delta y_0 = 42$, $\Delta^2 y_0 = 9$, $\Delta^3 y_0 = -25$, $\Delta^4 y_0 = 37$

Taking $x_0 = 40$, $x = 45$, $h = 10$

We have

$x = x_0 + ph$

or, $45 = 40 + p10$

or, $p = 0.5$

Using Newton's forward interpolation formula, we get,

$$y(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y(45) = 31 + 0.5(42) + \frac{0.5(0.5-1)}{2!}(9) + \frac{0.5(0.5-1)(0.5-2)}{3!}(-25) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(37)$$

$= 31 + 21 - 1.125 + 1.5625 - 1.4453$

$= 47.87$

Therefore $y(45) = 47.87$ The no of students with marks less than 45 is 47.87, that is 48. But the no of students with marks less than 40 is 31. Hence the no of students getting marks between 40 and 45 is = 48 – 31 = 17 students.

## 5. Question) The population of a certain town is show in the following table

| Year x | 1931 | 1941 | 1951 | 1961 | 1971 |
|---|---|---|---|---|---|
| Population y | 40.62 | 60.80 | 79.95 | 103.56 | 132.65 |

**Answer :-** Here h = 10,

Since the rate if the growth of the population is $dy/\ dx$ ,

We have to find $dy\ dx$ at x = 1961, which lies nearer to the end value if the table.

Hence we choose the origin at x = 1971 and we use Newton backward interpolation formula for derivative where,

**P =** $\frac{x-x0}{10}$ **=** $\frac{1961-1971}{10}$ **=-1**

The backward difference table

| Year (x) | Population (y) | $\nabla$y | $\nabla^2$y | $\nabla$ 3y | $\nabla$ 4y |
|---|---|---|---|---|---|
| **1931** | **40.62** | | | | |
| | | **20.18** | | | |
| **1941** | **60.80** | | **-1.03** | | |
| | | **19.15** | | **5.49** | |
| **1951** | **79.95** | | **4.46** | | **-4.47** |
| | | **23.61** | | **1.02** | |
| **1961** | **103.56** | | **5.48** | | |
| | | **29.09** | | | |
| **1971** | **132.65** | | | | |

So, by Newton interpolation formula for derivative we have

$$\frac{dy}{dx} = [\ \nabla y_4 + \frac{(2p+1)}{2}\ \nabla^2 y_4 + \frac{(3p^2+6p+2)}{2}\ \nabla^3 y_4 + \frac{(2p^3+9p^2+11p+3)}{2}\ \nabla^4 y_4 + \ldots\ldots\ ]$$

$$\left(\frac{dy}{dx}\right)_{p=-1} \frac{1}{10}\ [\ 29.09-\left(\frac{1}{2}\right)(5.48) + \frac{3(-1)^2+6(-1)+2}{6}\times 1.02 + \frac{2(-1)^3+9(-1)^2+11(-1)+3}{6}(4.47)\ ]$$

- ⇨ $\frac{1}{10}$ [29 - 2.74 - 0.17 + 0.3725]
- ⇨ $\mathbf{2.65}$

## 6. Question) Use fourth order Runge-Kutta method to fined y at x = 0.1 , given

**that** $\frac{dy}{dx}$ **= 3e$^x$ +2y, y(o) = 0 and h = 0.1**

**Answer :-** The fourth order Runge-Kutta formula is defined by

$y_i + 1 = y_i + 1/6\ [K_1 + 2K_2 + 2K_3 + K_4]$,

where i = 0, 1, 2, 3, ….

Where,

$$K_1 = hf(x_i, y_i)$$

$$K_2 = hf(x_i + h/2, y_i + k_1/2)$$

$$K_3 = hf(x_i + h/2, y_i + k_2/2)$$

$$K_4 = hf(x_i + h, y_i + K_3)$$

This method is most commonly used and is often referred to as Runge-Kutta method only. The error of the method is of order $O(h^5)$.

Given $f(x, y) = 3e^x + 2y$, $x_0 = 0$, $y_0 = 0$ and $h = 0.1$.

$K_1 = hf(x_0, y_0) = (0.1)\ f(0, 0) = (0.1)\ [3_e0 + 2 \times 0] = 0.3$.

$K_2 = hf(x_0 + h/2, y_0 + k_1/2\ ) = (0.1)f(0.05, 0.15)$

$$= (0.1)[3e^{0.05}+2(0.15)] = 0.3454$$

$K_3 = hf(x_0 + h/2, y_0 + k_2/2) = (0.1)\ f(0.05, 0.1727)$

$$= (0.1)[3e^{0.05}+2(0.1727)] = 0.3499.$$

$K_4 = hf\ (x0 + h, y0 + K3) = (0.1)\ f(0.1, 0.3499)$

$$= (0.1)[3e^{0.1}+2(0.3499)] = 0.4015.$$

Substituting these values in $y(x_0+h)$

$$= y_0 + 1/6\ [K_1 + 2K_2 + 2K_3 + K_4],$$

we get $y(0.1)$

$$= 0 + 1/6\ [0.3 + 2(0.3454) + 2\ (0.3499) + 0.4015]$$

$$= 0.3487$$

$\therefore\ y(0.1) = 0.3487$